Chapter review 2

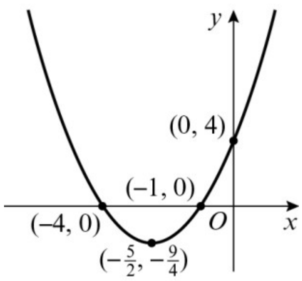
1 a $y^2 + 3y + 2 = 0$
$(y + 1)(y + 2) = 0$
$y = -1$ or $y = -2$

b $3x^2 + 13x - 10 = 0$
$(3x - 2)(x + 5) = 0$
$x = \frac{2}{3}$ or $x = -5$

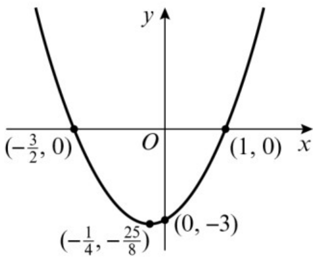
c $5x^2 - 10x = 4x + 3$
$5x^2 - 14x - 3 = 0$
$(5x + 1)(x - 3) = 0$
$x = -\frac{1}{5}$ or $x = 3$

d $(2x - 5)^2 = 7$
$2x - 5 = \pm\sqrt{7}$
$2x = 5 \pm \sqrt{7}$
$x = \dfrac{5 \pm \sqrt{7}}{2}$

2 a $y = x^2 + 5x + 4$
As $a = 1$ is positive, the graph has a $\cup$ shape and a minimum point.
When $x = 0$, $y = 4$, so the graph crosses the y-axis at (0, 4).
When $y = 0$,
$x^2 + 5x + 4 = 0$
$(x + 1)(x + 4) = 0$
$x = -1$ or $x = -4$, so the graph crosses the x-axis at $(-1, 0)$ and $(-4, 0)$.
Completing the square:
$x^2 + 5x + 4 = \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 4$
$= \left(x + \frac{5}{2}\right)^2 - \frac{9}{4}$
So the minimum point is at $\left(-\frac{5}{2}, -\frac{9}{4}\right)$.

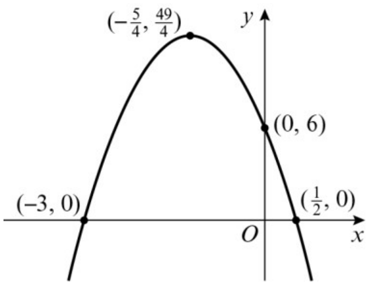


2 b $y = 2x^2 + x - 3$
As $a = 2$ is positive, the graph has a $\cup$ shape and a minimum point.
When $x = 0$, $y = -3$, so the graph crosses the y-axis at $(0, -3)$.
When $y = 0$,
$2x^2 + x - 3 = 0$
$(2x + 3)(x - 1) = 0$
$x = -\frac{3}{2}$ or $x = 1$, so the graph crosses the x-axis at $\left(-\frac{3}{2}, 0\right)$ and (1, 0).
Completing the square:
$2x^2 + x - 3 = 2\left(x^2 + \frac{1}{2}x\right) - 3$
$= 2\left(\left(x + \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right) - 3$
$= 2\left(x + \frac{1}{4}\right)^2 - \frac{25}{8}$
So the minimum point is at $\left(-\frac{1}{4}, \frac{25}{8}\right)$.



c $y = 6 - 10x - 4x^2$
As $a = -4$ is negative, the graph has a $\cap$ shape and a maximum point.
When $x = 0$, $y = 6$, so the graph crosses the y-axis at (0, 6).
When $y = 0$,
$6 - 10x - 4x^2 = 0$
$(1 - 2x)(6 + 2x) = 0$
$x = \frac{1}{2}$ or $x = -3$, so the graph crosses the x-axis at $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$.
Completing the square:
$6 - 10x - 4x^2 = -4x^2 - 10x + 6$
$= -4\left(x^2 + \frac{5}{2}x\right) + 6$
$= -4\left(\left(x + \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right) + 6$
$= -4\left(x + \frac{5}{4}\right)^2 + \frac{49}{4}$

2 c So the maximum point is at $\left(-\frac{5}{4}, \frac{49}{4}\right)$.



d $y = 15x - 2x^2$

As $a = -2$ is negative, the graph has a $\frown$ shape and a maximum point.

When $x = 0$, $y = 0$, so the graph crosses the y-axis at (0, 0).

When $y = 0$,

$15x - 2x^2 = 0$

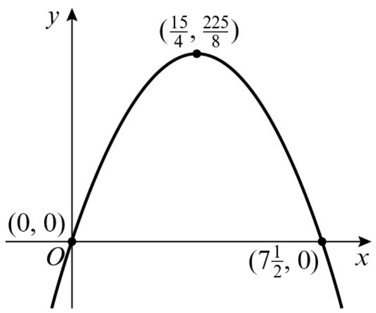
$x(15 - 2x) = 0$

$x = 0$ or $x = 7\frac{1}{2}$, so the graph crosses the x-axis at (0, 0) and $\left(7\frac{1}{2}, 0\right)$.

Completing the square:

$$15x - 2x^2 = -2x^2 + 15x$$
$$= -2\left(x^2 - \frac{15}{2}x\right)$$
$$= -2\left(\left(x - \frac{15}{4}\right)^2 - \left(\frac{15}{4}\right)^2\right)$$
$$= -2\left(x - \frac{15}{4}\right)^2 + \frac{225}{8}$$

So the maximum point is at $\left(\frac{15}{4}, \frac{225}{8}\right)$.



3 a $f(3) = 3^2 + 3(3) - 5 = 13$

$g(3) = 4(3) + k = 12 + k$

$f(3) = g(3)$

$13 = 12 + k$

$k = 1$

3 b
$$x^2 + 3x - 5 = 4x + 1$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$x = 3$ or $x = -2$

4 a $k^2 + 11k - 1 = 0$

$a = 1$, $b = 11$ and $c = -1$

Using the quadratic formula:

$$k = \frac{-11 \pm \sqrt{11^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-11 \pm \sqrt{125}}{2}$$

So $k = 0.0902$ or $k = -11.1$

b $2t^2 - 5t + 1 = 0$

$a = 2$, $b = -5$ and $c = 1$

Using the quadratic formula:

$$t = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{5 \pm \sqrt{17}}{4}$$

So $t = 2.28$ or $t = 0.219$

c $10 - x - x^2 = 7$

$\Rightarrow x^2 + x - 3 = 0$

$a = 1$, $b = 1$ and $c = -3$

Using the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-3)}}{2 \times 1}$$
$$= \frac{-1 \pm \sqrt{13}}{2}$$

So $x = -2.30$ or $x = 1.30$

d
$$(3x - 1)^2 = 3 - x^2$$
$$9x^2 - 3x - 3x + 1 = 3 - x^2$$
$$10x^2 - 6x - 2 = 0$$

$a = 10$, $b = -6$ and $c = -2$

Using the quadratic formula:

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(10)(-2)}}{2(10)}$$
$$= \frac{6 \pm \sqrt{116}}{20}$$

So $x = 0.839$ or $x = -0.239$

5 a $x^2 + 12x - 9 = (x + 6)^2 - 36 - 9$
$= (x + 6)^2 - 45$
$p = 1$, $q = 6$ and $r = -45$

b $5x^2 - 40x + 13 = 5(x^2 - 8x) + 13$
$= 5((x - 4)^2 - 16) + 13$
$= 5(x - 4)^2 - 67$
$p = 5$, $q = -4$ and $r = -67$

c $8x - 2x^2 = -2x^2 + 8x$
$= -2(x^2 - 4x)$
$= -2((x - 2)^2 - 4)$
$= -2(x - 2)^2 + 8$
$p = -2$, $q = -2$ and $r = 8$

d $3x^2 - (x + 1)^2 = 3x^2 - (x^2 + x + x + 1)$
$= 2x^2 - 2x - 1$
$= 2(x^2 - x) - 1$
$= 2\left(\left(x-\frac{1}{2}\right)^2-\frac{1}{4}\right)-1$
$= 2\left(x-\frac{1}{2}\right)-\frac{3}{2}$
$p = 2$, $q = -\frac{1}{2}$ and $r = -\frac{3}{2}$

6 $5x^2 - 2x + k = 0$
$a = 5$, $b = -2$ and $c = k$
For exactly one solution, $b^2 - 4ac = 0$
$(-2)^2 - 4 \times 5 \times k = 0$
$4 - 20k = 0$
$4 = 20k$
$k = \frac{1}{5}$

7 a $3x^2 + 12x + 5 = p(x + q)^2 + r$
$3x^2 + 12x + 5 = p(x^2 + 2qx + q^2) + r$
$3x^2 + 12x + 5 = px^2 + 2pqx + pq^2 + r$
Comparing x^2: $p = 3$ **(1)**
Comparing x: $2pq = 12$ **(2)**
Comparing constants: $pq^2 + r = 5$ **(3)**
Substitute (**1**) into (**2**):
$2 \times 3 \times q = 12$
$q = 2$
Substitute $p = 3$ and $q = 2$ into (**3**)
$3 \times 2^2 + r = 5$
$12 + r = 5$
$r = -7$
So $p = 3$, $q = 2$ and $r = -7$

b $3x^2 + 12x + 5 = 0$
$3(x+2)^2 - 7 = 0$
$3(x+2)^2 = 7$
$(x+2)^2 = \frac{7}{3}$

7 b $x+2 = \pm\sqrt{\frac{7}{3}}$
So $x = -2 \pm \sqrt{\frac{7}{3}}$

8 a $2^{2x} - 20(2^x) + 64 = (2^x)^2 - 20(2^x) + 64$
$= (2^x - 16)(2^x - 4)$

b $\text{f}(x) = (2^x - 16)(2^x - 4)$
Then either $2^x = 16 \Rightarrow x = 4$
or $2^x = 4 \Rightarrow x = 2$
$x = 2$ or $x = 4$

9 $2(x + 1)(x - 4) - (x - 2)^2 = 0$
$2(x^2 - 3x - 4) - (x^2 - 4x + 4) = 0$
$2x^2 - 6x - 8 - x^2 + 4x - 4 = 0$
$x^2 - 2x - 12 = 0$
$a = 1$, $b = -2$, $c = -12$
Using the quadratic formula:
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{52}}{2}$$
$$= \frac{2 \pm \sqrt{4 \times 13}}{2}$$
$$= \frac{2 \pm 2\sqrt{13}}{2}$$
So $x = 1 \pm \sqrt{13}$

10 $(x - 1)(x + 2) = 18$
$x^2 + x - 2 = 18$
$x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x = -5$ or $x = 4$

11 a The springboard is 10 m above the water, since this is the height at time 0.

b When $h = 0$, $5t - 10t^2 + 10 = 0$
$-10t^2 + 5t + 10 = 0$
$a = -10$, $b = 5$ and $c = 10$
Using the quadratic formula:
$$t = \frac{-5 \pm \sqrt{5^2 - 4(-10)(10)}}{2(-10)}$$
$$= \frac{-5 \pm \sqrt{425}}{-20}$$

11 b $t = -0.78$ or $t = 1.28$ (to 3 s.f.)
t cannot be negative, so the time is 1.28 seconds.

c $-10t^2 + 5t + 10$
$= -10(t^2 - 0.5t) + 10$
$= -10((t-0.25)^2 - 0.0625) + 10$
$= 10.625 - 10(t-0.25)^2$
$A = 10.625$, $B = 10$ and $C = 0.25$

d The maximum height is when $t - 0.25 = 0$, therefore when $t = 0.25$ s, $h = 10.625$ m.

12 a $f(x) = 4kx^2 + (4k + 2)x + 1$
$a = 4k$, $b = (4k + 2)$ and $c = 1$
$b^2 - 4ac = (4k + 2)^2 - 4 \times 4k \times 1$
$= 16k^2 + 8k + 8k + 4 - 16k$
$= 16k^2 + 4$

b $16k^2 + 4$
$k^2 \geqslant 0$ for all values of k, therefore
$16k^2 + 4 > 0$
As $b^2 - 4ac = 16k^2 + 4 > 0$, f($x$) has two distinct real roots.

c When $k = 0$,
$f(x) = 4(0)x^2 + (4(0) + 2)x + 1 = 2x + 1$
$2x + 1$ is a linear function with only one root, so f(x) cannot have two distinct real roots when $k = 0$.

13 $x^8 - 17x^4 + 16 = 0$
$(x^4)^2 - 17(x^4) + 16 = 0$
$(x^4 - 1)(x^4 - 16) = 0$
Then either $x^4 = 1 \Rightarrow x = \pm 1$
or $x^4 = 16 \Rightarrow x = \pm 2$
So $x = -2$, $x = -1$, $x = 1$ or $x = 2$

Challenge

a $\frac{a}{b} = \frac{b}{c}$
$\frac{b+c}{b} = \frac{b}{c}$
$b^2 - bc - c^2 = 0$
Using the quadratic formula:
$$b = \frac{-(c) \pm \sqrt{(-c)^2 - 4(1)(-c^2)}}{2(1)}$$
$$= \frac{c \pm \sqrt{5c^2}}{2}$$
$$= \frac{c \pm c\sqrt{5}}{2}$$
So $b : c = \frac{c \pm c\sqrt{5}}{2} : c$
Dividing by c:
$\frac{1 \pm \sqrt{5}}{2} : 1$
The length cannot be negative so
$b : c = \frac{1 + \sqrt{5}}{2} : 1$

b Let $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$
So $x = \sqrt{1 + x}$
Squaring both sides:
$x^2 = 1 + x$
$x^2 - x - 1 = 0$
Using the quadratic formula:
$$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{5}}{2}$$
The square root cannot be negative so
$$\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}} = \frac{1 + \sqrt{5}}{2}$$